MATH 2250 Calculus I
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Name: Answers
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Practice Quiz No. 2

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 State the (informal) definition of a limit. (Careful, it's surprising easy to make a logical mistake when rephrasing this definition.)

$\lim_{x \to a} f(x) = L$ if the values of $f(x)$ get closer and closer to L as the values of x get closer and closer to a.

Problem 2 Find the average rate of change of the function $f(x) = 2x + 4$ on the interval $[-1, 3]$.

$$(\text{Average rate of change}) = \frac{f(3) - f(-1)}{3 - (-1)}$$

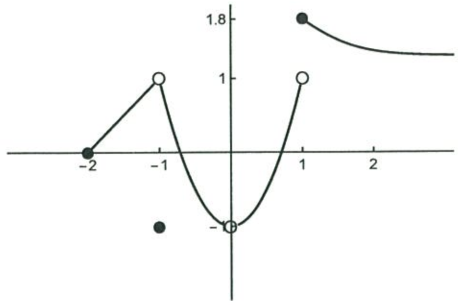
$$= \frac{2(3) + 4 - (2(-1) + 4)}{3 + 1}$$

$$= \frac{10 - 2}{4}$$

$$= \frac{8}{4}$$

$$= 2$$

Problem 3 Use the below graph to answer this question:



a *Find* $\lim_{x \to -1} f(x)$. $= 1$

b *Find* $\lim_{x \to 1^+} f(x)$. $= 1.8$

c *Find* $\lim_{x \to 1^-} f(x)$. $= 1$

d *Find* $\lim_{x \to 1} f(x)$. $=$ DNE

e *Is $f(x)$ continuous at $x = 0$? Why or why not?* No, because $f(0)$ does not exist

f *Is $f(x)$ continuous at $x = -1$? Why or why not?* No, because $\lim_{x \to -1} f(x) = 1 \neq -1 = f(-$

g *Is $f(x)$ continuous at $x = -2$? Why or why not?* Yes, because $\lim_{x \to -2^+} f(x) = f(-2) = 0$

h *Is $f(x)$ continuous? Why or why not?*

No, because there is at least one point in the domain of $f(x)$ where f is not continuous, for example, $x = -1$.

Problem 4 Evaluate:

$$\lim_{x\to 0} 5x^2 + 10x + \sin(x) + e^x$$

$$\underbrace{5x^2 + 10x}_{} + \underbrace{\sin(x)}_{} + \underbrace{e^x}_{}$$

Polynomial, so continuous. Trig function, so continuous. Exponential function, so continuous.

Remember, if $f(x)$ and $g(x)$ are continuous, then so is $f(x) + g(x)$. So $5x^2 + 10x + \sin(x) + e^x$ is continuous, so $\lim_{x\to 0} 5x^2 + 10x + \sin(x) + e^x = 5(0)^2 + 10(0) + \sin(0) + e^0 = 1$

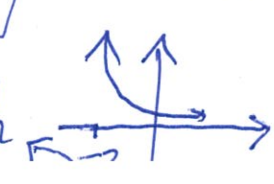
Problem 5 Evaluate:

$$\lim_{x\to -2} \frac{x-3}{x^2 - x - 6}$$

$\frac{x-3}{x^2-x-6}$, this is a rational function, so it is continuous. However, $(-2)^2 - (-2) - 6 = 0$, so $x = -2$ is not in the domain of $f(x) = \frac{x-3}{x^2-x-6}$

To find the limit, we factor:

$$\frac{x-3}{(x-3)(x+2)} = \frac{1}{x+2} \quad \text{assuming } x \neq 3$$

Now we can see from the graph  that the

Problem 6 Find the slope of the tangent line (i.e. the instantaneous rate of change) for the function $f(x) = x + 1$ at the point $a = 2$, i.e. find

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)+1 - (x+1)}{h}$$

$$= \lim_{h \to 0} \frac{x+h+1-x-1}{h}$$

$$= \lim_{h \to 0} \frac{h}{h}$$

$$= \lim_{h \to 0} 1 = 1$$, (notice we didnt use $a=2$)

Problem 7 Find the slope of the tangent line (i.e. the instantaneous rate of change) for the function $f(x) = \sqrt{2x+4}$ at the point $a = 1$, i.e. find

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2(1+h)+4} - \sqrt{2(1)+4}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2(1+h)+4} - \sqrt{6}}{h} \left(\frac{\sqrt{2(1+h)+4} + \sqrt{6}}{\sqrt{2(1+h)+4} + \sqrt{6}} \right)$$

$$= \lim_{h \to 0} \frac{(2(1+h)+4) + \cancel{\sqrt{6}\sqrt{2(1+h)+4}} - \cancel{\sqrt{6}\sqrt{2(1+h)+4}} - (6)}{h\left(\sqrt{2(1+h)+4} + \sqrt{6}\right)}$$

$$= \lim_{h \to 0} \frac{2 + 2h + 4 - 6}{h\left(\sqrt{2(1+h)+4} + \sqrt{6}\right)} = \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2(1+h)+4} + \sqrt{6}\right)} = \frac{2}{\sqrt{6}+\sqrt{6}}$$

Problem 8 Find the slope of the tangent line (i.e. the instantaneous rate of change) for the function $f(x) = \frac{1}{-5x+2}$ at the point $a = 0$, i.e. find

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{-5(0+h)+2} - \frac{1}{-5(0)+2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{-5h+2} - \frac{1}{2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2}{2(-5h+2)} - \frac{(-5h+2)}{2(-5h+2)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2+5h-2}{2(-5h+2)}}{h}$$

$$= \lim_{h\to 0} \frac{5\cancel{h}}{\cancel{h}(2)(-5h+2)}$$

$$= \lim_{h\to 0} \frac{5}{2(-5h+2)}$$

$$= \frac{5}{4}$$

Problem 9 Find the slope of the tangent line (i.e. the instantaneous rate of change) for the function ~~$f(x) = \sqrt{3x^2+1}$~~ at the point $a = 1$, i.e. find

Correct the problem: $f(x) = 3x^2+1$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{3(x+h)^2+1 - (3x^2+1)}{h}$$

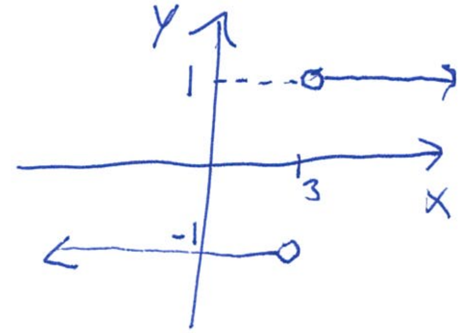
(I'm going to wait to plug in $a=1$)

$$= \lim_{h\to 0} \frac{1}{h}(3x^2+6xh+h^2+1-3x^2-1)$$

$$= \lim_{h\to 0} \frac{1}{h}(6xh+h^2)$$

$$= \lim_{h\to 0} (6x+h)$$

$= 6x$, now plug in $a=1 \Rightarrow 6(1) = 6 =$ (slope of tangent l



Problem 10 Evaluate:

$$\lim_{x\to 3^+} \frac{|x-3|}{x-3}$$

$x \to 3^+$, so $x > 3$, so $x - 3 > 0$

$\Rightarrow |x-3| = x-3$

$\Rightarrow \lim_{x\to 3^+} \frac{|x-3|}{x-3} = \lim_{x\to 3^+} 1 = 1$

Note that $\frac{|x-3|}{x-3} \neq 1$, because they aren't equal for $x = 3$, but they are equal for $x > 3$.

Problem 9 Evaluate:

$$\lim_{x\to 0} \frac{\sin(5x)}{3x}$$

$\lim_{x\to 0} \frac{\sin(5x)}{3x}\left(\frac{5/3}{5/3}\right)$

$= \lim_{x\to 0} \frac{5}{3}\left(\frac{\sin(5x)}{5x}\right)$

$= \frac{5}{3}\lim_{x\to 0} \frac{\sin(5x)}{5x}$

$= \frac{5}{3}\lim_{5x\to 0} \frac{\sin(5x)}{5x}$, call $y = 5x$

$= \frac{5}{3}\lim_{y\to 0} \frac{\sin(y)}{y} = \frac{5}{3}(1) = \frac{5}{3}$